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1991 J. Phys.: Condens. Matter 3 5335

(http://iopscience.iop.org/0953-8984/3/28/008)

J. Phys.: Condens. Matter 3 (1991) 5335–5339. Printed in the UK

The de Haas–van Alphen effect in a marginal Fermi liquid

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Received 6 March 1991

Abstract. High field–low temperature modifications to the Lifshitz–Kosevich theory of the de Haas–van Alphen effect arise from the marginal Fermi-liquid hypothesis. Consequences of the hypothesis for de Haas–van Alphen frequencies and amplitudes are discussed.

Recent photoemission results in the normal state of some high temperature superconducting compounds (Arko *et al* 1989, Campuzano *et al* 1990, Imer *et al* 1989) have been interpreted as supporting the existence of a Fermi surface. Because normal state properties such as resistivity, optical properties and NMR relaxation rates are frequently anomalous, the precise character of the responsible Fermi liquid may be unusual. For this reason, a phenomenological model (Varma 1989, Varma *et al* 1989), which provides a unified interpretation of several normal state properties, has gained considerable attention. The model, referred to as the marginal Fermi-liquid (MFL) model, is based on the single hypothesis that there are bosonic spin and charge response functions of the Fermi liquid that have, in addition to a normal contribution, an anomalous contribution $\chi_A(k, \omega)$ where

$$\mathrm{Im}\,\chi_A(k,\omega)\begin{cases} = -N(0)\,\omega/T & \text{for } |\omega| < T \\ = -N(0)\,\mathrm{sgn}\,\omega & \text{for } |\omega| > T \end{cases} \tag{1}$$

which is frequency limited by a sharp cutoff, $|\omega_X|$, and which is assumed to be effectively independent of k. Here T is the temperature and $N(0)$ is the density of states at the Fermi level of the unperturbed system. The persistence of this unusual form for $\mathrm{Im}\,\chi_A(k, \omega)$ has been observed most recently in both neutron (Hayden *et al* 1991) and Raman (Slakey *et al* 1990) scattering experiments.

On the other hand, the conclusive experiments for determining the presence and properties of any Fermi surface are generally regarded to be those which exhibit magneto-oscillatory effects, in particular the de Haas–van Alphen (DHVA) effect. Indeed, DHVA results have played a significant role in characterizing the unusual Fermi-liquid nature of several heavy fermion alloys. It is, however, generally anticipated that non-ideal sample quality and the high magnetic fields required to reach H_{c2} will present even greater obstacles to successful DHVA experiments in high temperature superconductors than they do in heavy fermion alloys. It is conceivable, however, that DHVA oscillation can be observed in the superconducting mixed state (Graebner and Robbins 1976). In spite of such obstacles, recent reports claim to have observed DHVA oscillations in very high fields in the superconducting compound $YBa_2Cu_3O_7$ (Kido *et al* 1990, Smith *et al* 1990). Assuming that experiments actively seeking DHVA oscillations are at least at the observation threshold, speculations about some consequences of the MFL hypothesis for the DHVA effect seem worthwhile and are made here.

In the low-temperature quasiparticle limit the grand potential Ω is

$$\Omega = -\frac{1}{\beta}\mathrm{Tr}\sum_{\zeta_n}\ln(-G^{-1}(\zeta_n)) \tag{2}$$

with $G(\zeta_n)$ the one-particle Green's function, $\zeta_n = \mathrm{i}(2n+1)\pi T$ and $\beta = 1/T$. Here $k_B = \hbar = 1$. Assuming applicability of Luttinger's theory for the DHVA effect in interacting systems (Luttinger 1961), the standard Lifshitz–Kosevich (Lifshitz and Kosevich 1955) formula for the oscillatory part of the quasiparticle limit is modified to display the change in frequency and amplitude (Engelsberg and Simpson 1970, Wasserman and Bharatiya 1979) arising from the anomalous MFL bosonic function as

$$\Omega_{\mathrm{osc}} = -\frac{(m_B\omega_c)^{3/2}T}{\pi^2}\sum_{\substack{n=0\\ m=\pm 1/2}}\sum_{r=1}^{\infty}\frac{(-1)^r}{r^{3/2}}\cos\left(\frac{2\pi r[\mu + mb - \mathrm{Re}(X(\zeta_n))]}{\omega_c} - \frac{\pi}{4}\right)$$
$$\times\exp\left(-\frac{2\pi r}{\omega_c}[\zeta_n - \mathrm{Im}(X(\zeta_n))]\right)\exp\left(-\frac{2\pi^2 r T_D}{\omega_c}\right). \tag{3}$$

where r is the harmonic number, μ is the chemical potential, $b = \gamma\mu_B B$, with γ the electron g-factor, μ_B the Bohr magneton, B the magnetic field, and where $\omega_c = eB/m_0$, with m_0 the bare band mass. The normal part of $\chi(k, \omega)$ is assumed to be absorbed into the exchange-correlation terms of a density functional calculation. Although inessential to the result, a conventional Dingle temperature, T_D, is assumed to adequately describe 'impurity' scattering. Here $X(\zeta_n)$ is the analytically continued field-independent part of the self-energy defined in terms of the retarded self-energy $\Sigma(k, \omega')$ by

$$X(k, \zeta_n) = -\frac{1}{\pi}\int_{-\infty}^{\infty}\mathrm{d}\omega'\,\frac{\mathrm{Im}\,\Sigma(k,\omega')}{\mathrm{i}\zeta_n - \omega'}. \tag{4}$$

The Luttinger approach to the DHVA effect has been surprisingly successful for understanding some many-body aspects of the DHVA amplitude (Engelsberg and Simpson 1970) even in a few cases where applicability might be regarded with suspicion (Rasul 1989, Stamp 1987, Wasserman *et al* 1989).

Assuming that the simple form of (1), with its UV cutoff ω_X, fairly represents the MFL hypothesis, a self-energy $X(\zeta_n)$ can be calculated from (1). Then, using (3), inferences may be drawn about the DHVA frequency and amplitude and, in particular, an effective mass. Renormalization (Zimanyi and Bedell 1990) seems not to qualitatively affect conclusions from the fundamental MFL hypothesis.

Using spectral representations for $G(\zeta_n)$ and $\chi_A(k, \zeta_n)$, the self-energy arising from the single-loop exchange approximation is

$$X(\zeta_n) = \frac{g^2N(0)}{\pi}\int_{-\infty}^{\infty}\mathrm{d}\omega'\,\mathrm{Im}\,\chi_A(k,\omega')\int_{-\infty}^{\infty}\mathrm{d}\omega\left(\frac{n(\omega') + 1 - f(\omega)}{\omega' + \omega - \mathrm{i}\zeta_n}\right) \tag{5}$$

where g is the total coupling constant for spin and charge channels with $n(\omega)$ and $f(\omega)$

the Bose and Fermi functions, respectively. The retarded self-energy, which can be approximated from (5) (Varma *et al* 1989) as

$$\Sigma(k, \omega) \sim g^2 N(0)^2[\omega \ln(x/\omega_X) - \mathrm{i}(\pi/2)x] \tag{6}$$

where $x = \max(|\omega|, T)$, is the basis for discussion of the conductivity and specific heat of the MFL. In particular, (6) yields a quasiparticle weight

$$z_k = (1 - \partial \operatorname{Re} \Sigma(\omega)/\partial\omega)^{-1}$$

which vanishes logarithmically at the Fermi surface, a feature that essentially defines marginality of the Fermi liquid.

The self-energy in (5) may be re-written as

$$X(\zeta_n) = -\frac{g^2 N(0)}{\pi}\int_0^\infty \mathrm{d}\omega' \operatorname{Im}\chi_{\mathrm{A}}(k, \omega') \int_{-\infty}^\infty \mathrm{d}\omega \left(\frac{n(\omega') + 1 - f(\omega)}{\mathrm{i}\zeta_n - (\omega' + \omega)} + \frac{n(\omega') + f(\omega)}{\mathrm{i}\zeta_n - (\omega - \omega')}\right) \tag{7}$$

where the ω-integration is done by closing the contour in the upper half ω-plane to give the imaginary quantity

$$X(\zeta_n) = 2\mathrm{i}Tg^2 N(0) \int_0^\infty \mathrm{d}\omega' \frac{\operatorname{Im}\chi_{\mathrm{A}}(k, \omega')}{\omega'}\left[1 + 2\sum_{\nu=1}^{n}\left(\frac{\omega'^2}{\omega'^2 + (2\pi\nu T)^2}\right)\right]. \tag{8}$$

Referring to (3), the absence of a real part to $X(\zeta_n)$ implies that the DHVA frequency deriving from the Fermi surface that forms an electron basis for the marginal hypothesis ought to be unaffected by the anomalous MFL response function.

Modifications to the amplitude are less benign. Ignoring impurity scattering, the DHVA amplitude A is, from (3)

$$A = \sum_{n=0}^{\infty} \exp\left(-\frac{2\pi r}{\omega_c}[\zeta_n - \operatorname{Im}(X(\zeta_n))]\right). \tag{9}$$

At high temperatures and weak fields the sum converges sufficiently rapidly for the amplitude to be approximated by the first term ($n = 0$) to obtain the Lifshitz–Kosevich form. Substituting from (1), one has

$$\operatorname{Im} X(\zeta_{n=0}) = 2Tg^2 N(0) \int_0^\infty \mathrm{d}\omega' \frac{\operatorname{Im}\chi_{\mathrm{A}}(k, \omega')}{\omega'} = -2Tg^2 N(0)^2[1 + \log(\omega_X/T)] \tag{10}$$

so the leading contribution to the amplitude is

$$A_{n=0} = \exp(-2\pi^2 T/\omega_c^*) \tag{11}$$

where $\omega_c^* = eB/m^*$, with m^* the enhanced mass

$$m^* = m_0\{1 + (2/\pi)g^2 N(0)^2[1 + \log(\omega_X/T)]\}. \tag{12}$$

This mass enhancement is generally expected to correspond to the specific-heat effective mass, however the result in (12) differs slightly from C_v given by Varma (1989). If a DHVA amplitude, A_0, is defined in terms of the effective mass of (12) according to the Lifshitz–Kosevich form, $A_0 = [2\sinh(2\pi^2 T/\omega_c^*)]^{-1}$, then with decreasing temperature, ($T < \omega_X$) the increasing effective mass of the MFL causes an amplitude suppression when

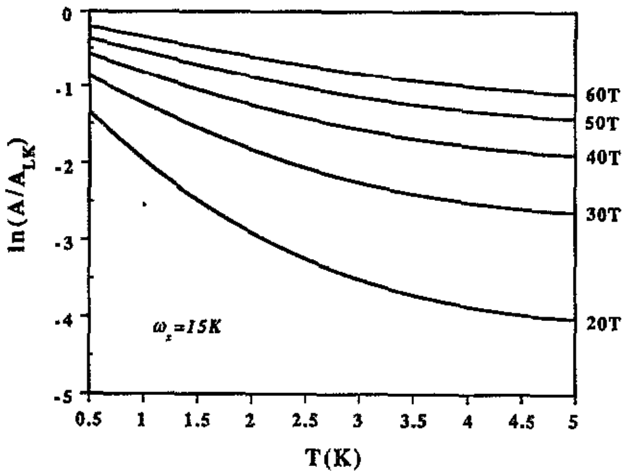

Figure 1. $\ln(A/A_{LK})$ plotted against temperature for indicated values of the magnetic field. The amplitudes A and A_{LK} are defined in the text. Parameters are $g^2N(0)^2 = \pi$, $\omega_X = 15$ K and $m_0 = 0.5$.

compared with a Lifshitz–Kosevich amplitude $A_{LK} = [2\sinh(2\pi^2 T/\omega^*_{LK})]^{-1}$, which has been defined in terms of a temperature-independent effective mass $m^*_{LK} = m_0[1 + (2/\pi)g^2N(0)^2]$ assumed to apply when $(T \geqslant \omega_X)$. The magnitude of the suppression is determined by the unknown parameters ω_X and $gN(0)$.

However, at very low temperature and very high fields the series of amplitudes in (9) converges slowly and many terms must be summed to obtain meaningful convergence. These additional $(n > 0)$ contributions in the series cause amplitude enhancements similar to those predicted (Engelsberg and Simpson 1970) and observed in Hg (Khalid *et al* 1988) for the electron–phonon interaction. As DHVA experiments will probably be done at $T < 2$ K and $B > 30$ T, estimates of total amplitudes could be of practical interest in assessing the experimental window for observing DHVA oscillations in a marginal Fermi liquid.

In figure 1 the temperature variation of the amplitude ratio $\ln(A/A_{LK})$, shown at several values of magnetic field indicates that there is a high-field–low-temperature recovery of some of the amplitude lost from the $\ln T$ term in m^*. The amplitude suppressions increase with increasing values of the cutoff parameter ω_X as shown in figure 2 for $B = 60$ T but with amplitude recovery at the lowest temperatures.

If high T_c materials in the normal state behave like marginal Fermi liquids described by (1) then the logarithmic temperature dependence of the effective mass exacts an amplitude penalty against observation of DHVA oscillations except at very high fields $B \geqslant 50$ T and very low temperatures $T \leqslant 1$ K where non-Lifshitz–Kosevich contributions effect some recovery of lost amplitude.

Acknowledgments

One of us (AW) wishes to thank the SERC and the Royal Society for supporting this work.

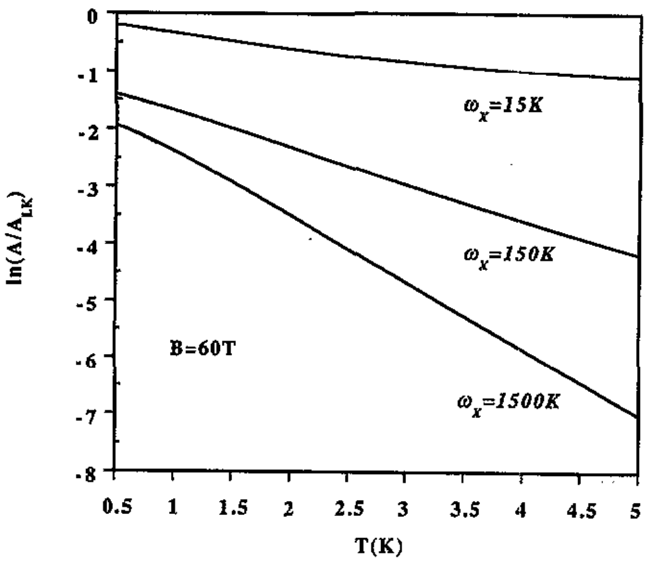

Figure 2. $\ln(A/A_{LK})$ plotted against temperature for $B = 60$ T for the indicated values of the cutoff parameter, ω_X, where $g^2N(0)^2 = \pi$ and $m_0 = 0.5$.

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